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Flux Motion through the Superconducting Surface Sheath

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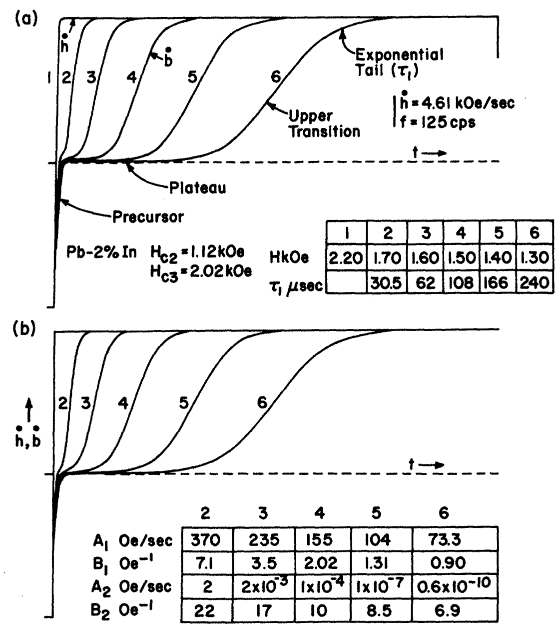
The response $\dot{b}$ of Pb-2% In alloy specimens in the superconducting surface-sheath state to a small ac field $\dot{h}$ with a triangular waveform has been studied. Features noted with this approach include a precursor, a plateau, and an exponential approach of $\dot{b}$ to $\dot{h}$. These observations and others point to the necessity of a more complete model for flux passage through the sheath. An analytical expression, based on a model of creep of flux spots in the sheath, is found to give an excellent quantitative description for Pb-2% In alloy specimens. The activation energy for the process is 10^{-3} eV $< U_0 < 10^{-2}$ eV and the activation distance is greater than 10 Å. This latter value is regarded as low but may result from geometrical effects within the sheath.

The response of the superconducting surface sheath to small low-frequency magnetic fields has been a subject of continued interest.[1] However, the interpretation of the data in terms of a single parameter, the critical current, derived on thermodynamic grounds[2] has been largely unsatisfactory. Although such models predict shielding currents of the observed order of magnitude, the sensitivity of the measured currents to slight changes in alignment of the sample with respect to the applied dc field, on the frequency of the ac field, and the dissipation mechanism responsible for the detected signal alike appear to be unexplained. Since the dynamic characteristics of these dependences are contained in the structure of the response waveform, we have carried out a careful study of the response of Pb-In alloy specimens to a small ac field with a triangular waveform. (This waveform is such that in any half-cycle the rate of change of the applied field is constant.) As a result, we have identified several systematic features characteristic of the dynamical penetration and expulsion of flux in these specimens, and, prompted by considerations of thermally activated creep of flux spots, have been able to find an analytical expression for the response. This flux-spot model provides an attractive explanation for our observations.

In Fig. 1(a) we show typical XY-recorder tracings of the response of a Pb-2% In sample to a 110 Hz triangular ac field superimposed on a dc field H, $H_{c2}<H<H_{c3}$, with all fields parallel to the axis of the cylindrical sample. These data were obtained using a PAR boxcar integrator in a standard arrangement. The tracing marked $\dot{h}$ is the time rate of change of the applied field while the others ($\dot{b}$) are the time rate of change of the field b within the specimen for five different values of the applied dc field H. The systematic features of the response are (1) a precursor at the start of each half-cycle, (2) a significant plateau, and (3) an exponential approach to $\dot{h}$. As H increases, the time constant τ_1 of the exponential tail decreases smoothly while the slope and height of the plateau increases. Eventually, the precursor and the tail merge into a slightly modulated exponential curve which gradually goes over, at H_{c3}, to $\dot{h}$, the normal-state response. Preliminary experiments on Nb-5% Mo alloys show that for those alloys the whole domain of the response waveform is in this "vanishing-plateau" regime.

In discussing the results of their dc investigations, Hart and Swartz[3] proposed the existence of quantized



	1	2	3	4	5	6
HkOe	2.20	1.70	1.60	1.50	1.40	1.30
τ_1 μsec		30.5	62	108	166	240

	2	3	4	5	6
A_1 Oe/sec	370	235	155	104	73.3
B_1 Oe^{-1}	7.1	3.5	2.02	1.31	0.90
A_2 Oe/sec	2	2×10^{-3}	1×10^{-4}	1×10^{-7}	0.6×10^{-10}
B_2 Oe^{-1}	22	17	10	8.5	6.9

FIG. 1. (a) Effect of a magnetic field on the specimen response. Note the reduction in time scales under increasing H for both the precursor and the exponential tail. (b) Corresponding model fits with values of the parameters. The precursor parameters here have large uncertainties.

flux spots in the surface sheath.[4] Even though in the present case (cylindrical wires 5.7 cm long, 1.7 mm diam) the surface in question is aligned carefully with the magnetic field, the alignment cannot be done sufficiently carefully (to within 10^{-10} rad) nor can the surface be made sufficiently smooth to avoid quantized penetration of magnetic flux through the sheath. We therefore expect that under these conditions the surface sheath would be populated with a considerable density of flux spots. As pointed out by Hart and Swartz, many of the properties of fluxoids in the mixed state may well pertain to flux spots. In particular, flux spots may be pinned by localized inhomogeneities, enter a critical state,[5] and move by a process of thermally activated creep.[6] Indeed, evidence for the observation of creep in the surface sheath has been reported by Beasley, Labusch, and Webb.[7] An extensive discussion of possible flux-spot effects in the surface sheath has been given by Park,[8] although it did not cover the specific situation we present.

We now note that when a flux spot moves a distance Δx the average field b changes by $\Delta b=\varphi_0\Delta x/\pi r^2 l$, where r and l are the radius and length of the cylinder, respectively, and φ_0 is the flux quantum. If we generalize to n flux spots per unit area moving with a velocity v, then $\dot{b}=2n\varphi_0 v/r$. We now have the basis of a model for $\dot{b}$ in terms of the velocity v and the density n.

Consideration of such models based on creep (in analogy with, for example, Anderson[6]) leads to a first expression for $v=Y\nu=Y\nu_0\ \exp[-(U_0-J\varphi_0 X)/kT]$ where Y is a distance moved per activation event occurring at a rate ν, ν_0 is a frequency factor, and $U_0-J\varphi_0 X$ is the simplest energy expression for flux spots pinned against a barrier of height U_0 and effective pinning length X by a Lorentz force $J\varphi_0$. This immediately gives

$$\dot{b}=(2n\varphi_0 Y\nu_0/r)\ \exp[-(U_0-J\varphi_0 X)/kT]$$
$$=A\ \exp[B(h-b)]. \quad (1)$$

The latter form arises from using Maxwell's equation to identify the surface-current density J with $(h-b)/4\pi$, where h is the applied field and b the internal field. This appears to be the simplest possible model and we remark that several somewhat more complicated models appear equally capable of leading to the exponential form. In the experiments described here, flux alternately leaves and enters the closed sheath, corresponding in a flux-spot model to reversals of the direction of movement. When the field reverses, the movement of flux spots in one direction has to be decreased and movement in the other direction increased. Thus an improved version of Eq. (1) will include two terms: one corresponding to the speeding up and one to the slowing down of flux spots. It is by no means clear that the creep constants A and B are the same for both processes and there is evidence[7] that in flux creep in mixed-state superconductivity below H_{c2}, considerable differences may exist between the constants in the two processes. Accordingly, our final model expression for $\dot{b}$ includes both terms and is

$$\dot{b}=A_1\exp[B_1(h-b)]-A_2\exp[-B_2(h-b)]. \quad (2)$$

For a given $h(t)$, such as the triangular waveform used in these experiments, this equation will in principle give $b(t)$. The precursor part of the waveform is

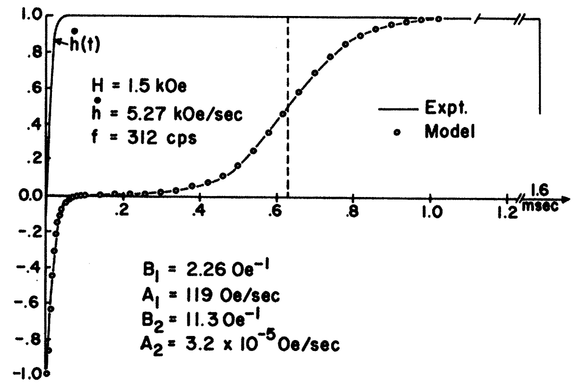


FIG. 2. Specimen response taken at a higher frequency where the precursor is more apparent and its parameters easier to estimate. In obtaining the model fit, the time variation of $\dot{h}$ was taken into account. The dashed line indicates the response expected under a critical-current model.

dominated by the second term in this expression, whereas the region beyond the plateau is dominated by the first term. In the present case, the marked plateau makes it relatively simple to determine the appropriate constants.

Figure 1(b) shows the model curves corresponding to the $\dot{b}$ curves of Fig. 1(a). At the frequency used, the precursor was not very marked and the A_2 and B_2 parameters have relatively large uncertainties. In order to make a somewhat more definitive test of Eq. (2), a higher frequency (at which the precursor is more prominent) was used, and the experimental curve together with the best fit of Eq. (2) are shown in Fig. 2. The agreement between experiment and the model is excellent and is well within the experimental limitations. The parameter most readily obtained from the experimental values for the constants is the activation distance X. From Eq. (1) we identify B as $\varphi_0 X/4\pi kT$, where the activation distance X will be $\sim$10 Å for $B=2.8\ \text{Oe}^{-1}$ at 4.2°K using $\varphi_0=2\times10^{-7}$ G cm^2. This distance corresponds in principle to the distance moved by one flux spot during the activation event. The value of 10 Å is smaller than seems reasonable if a flux spot is expected to have a minimum size of a coherence length, i.e., a few hundred angstroms. It could, however, be consistent if a substantial angle exists between the Lorentz force and the displacement X. Such a situation would result if the flux spots passed through the sheath at a considerable angle to the normal to the sheath. Using Eq. (1), it is possible to make an order-of-magnitude estimate of the pinning energy U_0. Taking $n\varphi_0=2$ Oe, $Y=1\ \mu$, and $\nu_0=10^{10}s^{-1}$, the experimental value of $A=120$ g/sec leads to $U_0=13kT\approx5\times10^{-3}$ eV. Variation of the parameters suggests that U_0 lies within the range $1.4\times10^{-3}\ \text{eV}<U_0<1.4\times10^{-2}$ eV. The excellent fit implies that, over the range studied here, creep is the dominant process. A flux-flow situation would be identified qualitatively by an asymmetric upper transition rather than the symmetric S-shaped transition seen in these specimens.

As can be seen from Fig. 1, Eq. (2) represents an excellent fit to the observed transition for applied fields H, varying all the way from H_{c2} to H_{c3}. Different experimental curves have been observed in Nb-5% Ti alloys, where the plateau is not so marked, and in Nb-5% Mo alloys, where the plateau essentially vanished. The four-parameter model (2) has proved capable of predicting qualitatively the observed forms, but our experiments have not been as detailed as for the Pb-8% In alloy and the extent of the quantitative fit is not yet known. Preliminary studies of the temperature dependence between 1.4 and 4.2°K and the frequency dependence up to 1 kHz have been carried out and again Eq. (2) accounts satisfactorily for the transitions observed. Although it is well known that properties of the surface sheath are sensitive to the state of the surface,[9] specific exploration of this point has been limited. We have noted in some specimens a more complicated response than those presented here. In these cases, at least two separate sets of constants apply in different parts of the waveform. This might be expected from a specimen with inhomogeneous pinning characteristics and we believe that homogeneity of the surface is an important factor in obtaining responses which fit the proposed equation (2).

Our calculations have been based on a model in which flux spots are pinned in the surface sheath in a manner similar to that postulated for the pinning of fluxoids in the bulk. We have made other observations on surface-sheath behavior which are consistent with this. These include some hysteresis on changing the applied field H and a phenomenon analogous to the peak effect.[10] The extreme sensitivity of the transition to sample alignment can be understood in the context of the flux-spot model. For example, the density and arrangement of flux spots can depend significantly on angle. An intrinsic frequency dependence results from Eq. (2) as a result of the time dependence allowed. Preliminary observations[11] suggest that the situation is somewhat more complicated in that the constants $A_{1,2}$ and $B_{1,2}$ do undergo some change with frequency, as can readily be seen from a comparison of Figs. 1 and 2. Finally, the sensitivity of the sheath response to the state of the surface clearly results from changes in the details of the pinning. Experiments exploring these and other points are currently underway.

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